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# Final Technical Report

AFOSR Grant F49620-95-1-0067

November 15, 1994, to November 14, 1997

## Summary

This project was concerned with the problem of determining the physical and geometric properties of an unknown inhomogeneity from a knowledge of its effect on a given time-harmonic electromagnetic wave with particular concern to non-destructive testing and medical imaging. The main accomplishments were 1) The discovery of a linear method for determining the support of aberrant inhomogeneities without any a priori assumptions on either the frequency or magnitude of the inhomogeneity, 2) The application of this new linear method to problems in microwave medical imaging, 3) The analysis and numerical implementation of a method of "perfectly matched layers" for the solution of Maxwell's equations, and 4) The derivation of an adaptive method for mesh refinement to produce a far field pattern of prescribed accuracy.

## Professional Personnel

Dr. David Colton    Unidel Professor of Mathematics  
                         University of Delaware  
                         Co-Principal Investigator

Dr. Peter Monk     Professor of Mathematics  
                         University of Delaware  
                         Co-Principal Investigator

## Report on Research

The research of AFOSR Grant F49620-95-1-0067 was concerned with the development of new methods to solve the inverse scattering problem for fixed frequency time harmonic electromagnetic waves. The main problem under consideration was the determination of the support of aberrant inhomogeneities from a knowledge of the scattered wave. Such inverse problems are particularly relevant for anisotropic media where the inverse problem is strongly non-unique and determining the support of an anomaly is in general the best that can be expected. Fortunately, for many problems in medical imaging and non-destructive testing, such information is all that is needed.

In order to effectively study and analyze a proposed algorithm for solving an inverse scattering problem, it is necessary to have accurate near-field or far-field scattering data at one's disposal. This leads to the problem of developing reliable numerical schemes for solving the direct scattering problem for Maxwell's equations. A particularly attractive method for doing this was the idea of "perfectly matched layers" as originally proposed by Bérenger. The original ideas of Bérenger were extended and mathematically analysed by Collino and Monk [4] during the period of this research effort. Such an analysis was complicated by the non-standard nature of the modified Maxwell equations in the layer. Finally, given an efficient method for solving the direct scattering problem for Maxwell's equations, one wishes to produce a far-field or near-field radiation pattern of prescribed accuracy and to this end Monk and Suli [23] have investigated an adaptive method for mesh refinement to produce such radiation patterns.

Given accurate far-field (or near-field) data, one can now analyze inversion algorithms for inverse scattering problems. To this end, during the period of this research effort, we have developed a new method (called the "simple method") for determining the *support* of aberrant inhomogeneities rather than the precise *values* of the index of refraction [11], [18]. In particular, the "simple method" mathematically places a grid on the material being probed, solves a *linear* integral equation of the first kind for each point on this grid, and then determines the support of the aberrant inhomogeneity from a knowledge of the solutions to the set of integral equations. Note that although the integral equations are linear, this is an exact method, i.e. no approximations have been made nor any assumptions on whether or not the frequency is high or low. The mathematical basis for this method rests

on showing the existence of a unique solution to a new class of boundary value problems for partial differential equations called interior transmission problems and the fact that this solution can be approximated by a Herglotz wave function.

In [16] and [17] we have used the "simple method" to study the problem of detecting tumors in the upper part of the lower leg and showed that the time for reconstruction was reduced from about three hours to fourteen seconds! Furthermore, it was shown in [13] that the "simple method" can also be used for certain types of anisotropic material, thus opening the door for the use of inverse scattering methods for materials of this type with particular reference to the study of the impact toughness of airplane canopies by non-destructive methods. The implications of this work for both medical imaging and non-destructive testing are obvious: if developed successfully this will be the first method for solving the inverse scattering problem that is rapid to implement but is independent of both ad hoc assumptions on the material and the frequency of the probing wave.

### Personnel Supported

- Faculty  
D. Colton, P. Monk (Principal Investigators)
- Post-Doctoral Students  
R. Potthast (supported by Deutsche Forschungsgemeinschaft)  
M. Piana (supported by Consiglio Nazionale delle Ricerche)
- Graduate Students  
C. Labreuche (supported by Thomson CSF-LCR)  
Thesis: Inverse Obstacle Scattering Based on Resonant Frequencies (University of Paris)
- S. Fadoulourahmane (supported by Academy of Finland)  
Thesis: An Inverse Problem for Time Harmonic Electromagnetic Waves in an Inhomogeneous Orthotropic Medium (University of Oulu)

## Publications

1. K. Chadan, D. Colton, L. Päivärinta and W. Rundell, *An Introduction to Inverse Scattering and Inverse Spectral Problems*, SIAM Monographs on Mathematical Modeling and Computation, SIAM, Philadelphia, 1997.
2. G. Cohen and P. Monk, Condensation de masse par quadrature de Gauss pour les equations de Maxwell, in *Méthodes Numériques d'Ordre Elve pour les Ondes en Régime Transitoire*, INRIA, Le Chesnay, 1995.
3. G. Cohen and P. Monk, Gauss point mass lumping schemes for Maxwell's equations, *Numerical Methods for Partial Differential Equations*, to appear.
4. F. Collino and P. Monk, The perfectly matched layer in curvilinear coordinates, *SIAM J. Scientific Computing*, to appear.
5. D. Colton, A survey of selected topics in inverse electromagnetic scattering theory, in *Inverse Problems in Wave Propagation*, G. Chavent, et. al. editors, Springer-Verlag, New York 1997, 105-127.
6. D. Colton, Qualitative methods in inverse scattering theory, in *Inverse Problems in Medical Imaging and Nondestructive Testing*, H. Engl, et. al. editors, Springer-Verlag, Vienna, 1997, 36-42.
7. D. Colton, The inverse scattering problem for an orthotropic medium, in *Modern Mathematical Methods in Diffraction Theory and its Applications in Engineering*, E. Meister, editor, Peter Lang, Frankfurt, 1997, 31-39.
8. D. Colton, A linear method for solving inverse scattering problems in the resonance region, in *Inverse Problems of Wave Propagation and Diffraction*, G. Chavent, et. al. editors, Springer-Verlag, Berlin, 1997, 86-92.

9. D. Colton, A brief introduction to inverse scattering theory, in *Integral Methods in Science and Engineering, Volume One: Analytic-Methods*, C. Constanda, et. al. editors, Longman Publishing, Essex, 1997, 3-8.
10. D. Colton and C. Erbe, Spectral theory of the magnetic far field operator in an orthotropic medium, in *Nonlinear Problems in Applied Mathematics*, T. S. Angell, et. al. editors, SIAM, Philadelphia, 1996, 96-103.
11. D. Colton and A. Kirsch, A simple method for solving inverse scattering problems in the resonance region, *Inverse Problems* 12 (1996), 383-393.
12. D. Colton, A. Kirsch and P. Monk, A new algorithm in electromagnetic inverse scattering theory with an application to medical imaging, *Mathematical Methods Applied Science* 20 (1997), 385-401.
13. D. Colton, R. Kress and P. Monk, Inverse scattering from an orthotropic medium, *J. Comp. Applied Math.* 81 (1997), 269-298.
14. D. Colton and P. Monk, The detection and monitoring of leukemia using electromagnetic waves: numerical analysis, *Inverse Problems* 11 (1995), 329-342.
15. D. Colton and P. Monk, A new approach to detecting leukemia: using computational electromagnetics, *IEEE Comp. Science Engineering* 2(1995), 46-52.
16. D. Colton and P. Monk, A linear sampling method for the detection of leukemia using microwaves, *SIAM J. Applied Math.*, to appear.
17. D. Colton and P. Monk, Mathematical problems in microwave medical imaging, in *Computational Radiology and Imaging: Therapy and Diagnostics*, Springer-Verlag, to appear.

18. D. Colton, M. Piana and R. Potthast, A simple method using Morozov's discrepancy principle for solving inverse scattering problems, *Inverse Problems*, 13 (1997), 1477-1493.
19. P. Monk, The near field to far field transformation, *COMPEL* 14 (1995), 41-56.
20. P. Monk, Sub-gridding FDTD schemes, *ACES Journal* 11 (1996), 37-46.
21. P. Monk, Mass lumping edge elements in three dimensions, in *Proceedings of the Third International Conference on Spectral and Higher Order Methods*, V. Il'in and R. Scott, editors, Houston Journal of Mathematics, 1996, 181-192.
22. P. Monk and E. Süli, Error estimates for Yee's method on non-uniform grids, *IEEE Trans. Magnetism* 30 (1994), 3200-3203.
23. P. Monk and E. Süli, The adaptive computation of far field patterns by a posteriori error estimation of linear functionals, submitted for publication.
24. P. Monk and O. Vacus, Error estimates for a numerical scheme for ferromagnetic problems, submitted for publication.
25. Y. Wang, P. Monk and B. Szabo, Computing cavity models using the p-version of the finite element method, *IEEE Trans. Magnetism* 32 (1996), 1934-1940.

### **Honors/Awards**

Professor Colton was appointed Unidel Professor of Mathematical Sciences at the University of Delaware effective February 12, 1996.